Review of the Development in Process Capability Analysis

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Abstract—— This review paper is devoted to the study for the analysis of the process capability of the manufacturing processes. The process capability indices C_p; C_{pk} C_{pm}, C_{pmk}, C_{py} and C_{pc} are presented, related to process parameters and the practical applications of the conventional as well as some new indices in the manufacturing industries are provided.

Keywords— Quality control, Process capability index, Non-normal distribution, Gamma and Weibull Distribution,Industrial application .

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1. INTRODUCTION

Process capability compares the process output with the customer's specification. Two parts of process capability are: i) Measure the variability of the output of a process, and ii) Compare that variability with requirement specification or product tolerance.

Process capability analysis is the evaluation of a production process to determine whether or not the inherent variability of its output falls within the acceptable range and process capability index or process capability ratio is a statistical measure of process capability. The concept of process capability holds meaning for processes that are in a state of statistical control. Process capability indices measure how much "natural variation" a process experiences relative to its specification limits and allows different processes to be compared with respect to how well an organization controls them.

A process capability index uses both the process variability and the process specifications to determine whether the process is capable. The process capability index (PCI) is a value which reflects real-time quality status. The PCI is considered as one of the quality measurements tool. In practice, process capability indices (PCIs) are used as a means of measuring process potential and performance. Moreover, most PCIs have been developed or investigated under the assumption that components have a lifetime with a normal distribution. In many processes, the quality characteristics may follow the non-normal distribution e.g. Weibull, Exponential, and Geometric distribution etc. In some cases, the characteristics of the product may be interrelated. It is necessary to develop the process capability measure for the quality characteristics related to the above mentioned distributions and sampling distribution of their estimate. Under non normal distribution, some properties of the PCIs and their estimators differ from those of normal distribution. To utilize the PCIs more reasonably and accurately in assessing the lifetime performance of components, this study is conducted.

2. PROCESS CAPABILITY INDICES FOR QUALITY CHARACTERISTICS FOLLOWING VARIOUS DISTRIBUTIONS

2.1. PCI For Normal Distribution

Process capability analysis is based on some fundamental assumptions that is, the process is stable and that the studied characteristic is normally distributed. There are several statistics that can be used to measure the capability of a process. According to the philosophy of the quality control approach, process capability indices of any process can be divided into capability indices of the first and second generation. The design of the first generation capability indexes (C_P, C_{pk}) is based on classical philosophy of the statistical process control. According to that philosophy all measurement results within required tolerance interval are intended to be good. Measurements outside tolerance interval are considered to be bad. Under these assumptions the two most widely used indices in industry are Cp and C_{pk}, where C_P was presented by Juran (1974) and C_{pk} by Kane (1986).

$$Cp = \frac{USL - LSL}{6\sigma} \qquad Cpk = \min\left\{\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right\}$$

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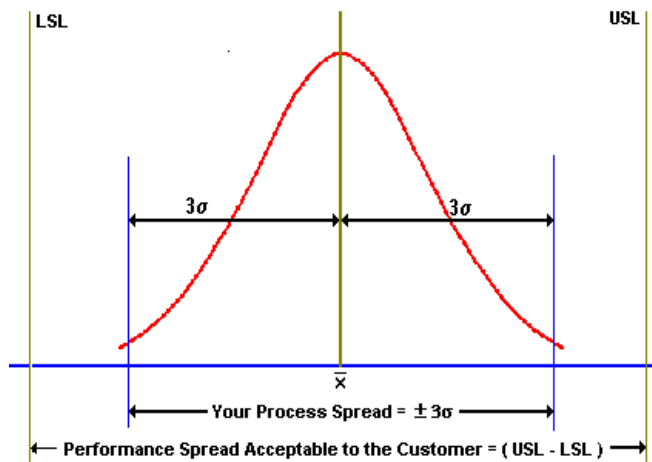
Fig.1 Normal distribution

Where [LSL, USL] is the specification interval, μ is the process mean and σ is the process standard deviation of the in-control process. Henceforth, we call the process mean μ and the process standard deviation σ for the process parameters. The capability index C_P relates the distance between the specifications limits to the range over which the process is actually varying.

2.2. PCI For Non Normal Distribution

Second generation capability index (C_{pm}) is rising from new approach to the quality improvement (Taguchi approach). It is not enough to know that measurements are so called good (being within the tolerance interval) but important is knowledge on how good they are. Such index enables to determine whether the values of the searched quality index approach to the tolerance limits even when all measurement results fit within the tolerance. Since the indices C_P and C_{pk} do not take into account of the differences between the processes mean and its target value, Chan (1988) and Pearn (1992) considered this difference to develop indices C_{pm} and C_{pmk} as follows:

$$Cpm = \left(\frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}} \right)$$

$$Cpm = \min\left\{ \frac{USL - \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{\mu - LSL}{3\sqrt{\sigma^2 + (\mu - T)^2}} \right\},$$

Where, T represents the target value of quality characteristics. The process parameters μ and σ^2 are estimated from the sample mean variance for $\bar{X}$ and S^2, when μ and σ^2 are unknown. Rodriguez (1992) proposed a process capability index (C_{pk}) for Lognormal distribution as

$$Cpk = Min\left(\frac{Ue^{-\xi} - 1}{e^{3\sigma} - 1}, \frac{1 - Le^{-\xi}}{1 - e^{-3\sigma}} \right)$$

And estimated proportion of non-conforming items as $\phi\{(\log L - \xi)/\hat{\sigma}\} + 1 - \phi(\log U - \xi)/\hat{\sigma}$ Where ξ = Scale parameter, $\hat{\sigma}$ = Estimated Standard deviation.

A superstructure or family of capability indices, containing C_p, C_Pk, C_{pm} and Cpmk involving two parameters, is introduced by Vannman (1993) as:

$$Cp(u,v) = \frac{d - u|\mu - m|}{3\sqrt{\{\sigma^2 + v(\mu - T)^2\}}}$$

Where $d = \frac{U - L}{2}$ and $m = \frac{U + L}{2}$

By letting u = 0 or 1 and v = 0 or 1 in the above equation, we can obtain four basic indices i.e. Cp(0,0) = Cp, C_P(1,0) = Cpk, C_P(0,1) = Cpm and C_P (1.1) = C_{pm}k. Chan and Mak (1993) propose a Process Capability indicator as:

$$Cl = \frac{|T - \mu|}{\left(\frac{U - L}{2}\right)} + w \frac{6\sigma}{(U - L)}, 0 \le w \le 1$$

= DI + w (VI)

Where DI= Deviation Indicator = $\frac{|T - m|}{\left(\frac{U - L}{2}\right)}$

VI = Variation Indicator = $\frac{6\sigma}{(U - L)} = \frac{1}{Cp}$

Most capability indexes assume that the quality characteristic is normally distributed. Clements (1989), Kotz and Johnson (1993), and Sommerville and Montgomery (1996) among others, comment in detail on the distortion in information provided by these indices when process distribution moves away from the assumed normal. Ahmad and saleh (1999) presented a generalization of Clements' method with asymmetric tolerances. These quantile-based indices, C_P and C_{pk} for non normal data are defined as follow:

$$Cp = \frac{USL - LSL}{X_{99.865} - X_{0.135}} \qquad Cpk = \min\left(\frac{USL - X_{50}}{X_{99.865} - X_{50}}, \frac{X_{50} - LSL}{X_{50} - X_{0.135}} \right)$$

Wang (1996), Zimmer (1998), Kan and Yasici (2006) and Mousa & Jaheen (2002) have presented a comprehensive review of Burr distribution and its application to many non-normal situations.Ahmed and Rahbar (2001) developed an

asymptotic theory for a capability index for arbitrary populations. Nahar and Zimmer (2001) and Chang(2002) considered the process capability indices for skewed populations. Abbasi (2006) used simulated annealing algorithm to estimate three parameters of Weibull distribution. Process capability analysis when observations are autocorrelated is investigated by Noorossana (2002) using time series modeling and regression analysis. Luceno (1996) suggested a process capability index which seems to be insensitive to departure from the assumption of normal variability. A detailed discussion and references on dealing with non-normality of the process distribution are given in Kotz and Johnson (2002).

2.3. PCI For Discrete Distribution

The most widely used such indices are C_p, C_{pk}, C_{pm}, and C_{pmk} or their generalizations for non-normal processes, suggested by Clements (1989), Pearn and Kotz (1994), and Pearn and Chen (1995). Often, however, one is faced with processes described by a characteristic whose values are discrete. Therefore, in such cases none of these indices can be used. Perakis and Xekalaki (2004) proposed a new index, which can be used regardless of whether the examined process is discrete or continuous. This index is defined as

$$Cpc = \frac{1 - p_0}{1 - p}$$

Where p and p_0 denote the proportion of conformance (yield) and the minimum allowable proportion of conformance of the examined process, respectively. As is well known, the term proportion of conformance refers to the probability of producing within the so-called specification area, i.e. the interval determined by L and U. If the tolerances are unilateral, then the value of p is given by P(X > L), if only L has been set, and by P(X < U), if only U has been assigned.

2.4. PCI for Gamma And Weibull Distribution

Since Motorola, Inc. introduced its Six Sigma quality initiative in the 1980s; quality practitioners have questioned why the followers of this initiative have added a 1.5σ shift to the process mean when estimating process capability. The advocates of Six Sigma have claimed that such an adjustment is necessary, but they have offered only personal experiences and empirical studies as justification for this claim (see Bender, 1975;Evans, 1975; Gilson, 1951). By examining the sensitivity of control charts to detect changes of various magnitudes, Bothe (2002) provided a statistically based reason for this claim. In his study, Bothe assumed that the process data is approximately normally distributed. However, non-normal processes occur frequently, in particular, in the semiconductor

industry. Pyzdek (1992) mentioned that the distributions of certain chemical processes, such as zinc plating in a hot-dip galvanizing process, are very often skewed. Choi et al. (1996) presented an example of a skewed distribution in the ''active area'' shaping stage of the wafer's production processes. Gamma distribution (skewed) covers a wide class of non-normal applications, including the manufacturing of semiconductor products, head/gimbals assembly for memory storage systems, jet-turbine engine components, flip-chips and chip-on-board, audio-speaker drivers, wood products, and many others.

The control charts are commonly used in many industries for providing early warning for the shift in the process mean. For example, the cumulative sum chart is known to be effective on detecting sustained shifts in the process mean (see e.g. Lucas and Crosier, 2000; Luceno and Puig-Pey, 2002; Lucas, 1976). If the control chart detects a process mean shift, then the process is not under control. However, for momentary process mean shifts, it may be beyond the control chart detection power. Consequently, the undetected shifts may result in overestimating process capability. If the process mean shifts are not detected, then unadjusted Cpk would overestimate the actual process yield. Bothe (2002) provided a statistical reason for considering such a shift in the process mean for normal processes. However, if the capability indices are based on the assumption of a normal distribution of data but are used to deal with non-normal observations, the values of the capability indices may, in the majority of situations, misrepresent actual product quality Ya-Chen Hsu, W.L. Pearn, Pei-Ching Wu (2007) examines Bothe's approach and finds that the detection power of the control chart is less than 0.5 when data comes from gamma distribution. This shows that Bothe's adjustments are inadequate when we have gamma processes. Then, the adjustments under various sample sizes (n) and gamma parameters (N) with a fixed detection power of 0.5 are calculated. Finally, the process capability formula is adjusted to accommodate the undetected shifts.

$$dynamic\ Cpk = \min\left\{\frac{USL - F_{0.5} - AS_{50}\sigma}{F_{0.99865} - F_{0.5}}, \frac{F_{0.5} - AS_{50}\sigma - LSL}{F_{0.5} - F_{0.00135}}\right\}$$

As a result, our adjustments provide significantly more accurate calculations of the capability of gamma processes. Ya-Chen Hsu, W. L. Pearn and Chun-Seng Lu (2011) calculate the mean shift adjustments under various sample sizes n and Weibull parameter, with the power fixed to 0.5. Then, we implement the adjustments to accurately estimate capability index Cpk for Weibull processes with mean shift consideration. With detection power of the Erto's-Weibull control chart fixed to 0.5, using the adjusted process capability formula, the engineers could determine the actual process capability more accurately.

2.5. PCI FOR POISSON DISCRETE DISTRIBUTION

Maiti(2011) have proposed a generalized process capability index that is the ratio of proportion of specification conformance to proportion of desired conformance.

$$C_{py} = p/p_0{}'$$

where p is the process yield that is F(U) –F(L),F(t) =P(X $\leq$ t) is the cumulative distribution function of X, and p0 is the desirable yield that is;

 p0 = F (UDL) – F (LDL). Bayesian estimation of the index has been considered under squared error loss function. Normal, exponential nonnormal and Poisson discrete processes have been taken into account.

2.6. PCI for Pearsonian Distribution

When the process has a distribution of Pearsonian type, W. L. Pearn and K. S. Chen proposed an estimator for non-normal Pearsonian populations (1995), $C_{Np}(u, v)$ by using Clements' method (Clements, 1989)as follows:

$$\hat{C}_{Np}(u,v) = \frac{d - u|\hat{M} - m|}{3\sqrt{\left(\frac{U_p - L_p}{6}\right)^2 + v(\hat{M} - T)^2}}$$

Where d = (USL±LSL)/2, where USL and LSL are respectively the upper and lower specification limits, m = (USL + LSL)/2, the specification center, and T is the target value and M is the median.

 Pearn and Kotz applied Clements' method to obtain first approximation estimators of PCIs for non-normal populations to the two more advanced PCIs, C_{pm} and C_{pmk} developed by Chan and Pearn.

3. INDUSTRIAL APPLICATION OF PCI

Some of the application areas are described below:-

* Predicting how well the process will hold tolerances (Manufacturing) - For various types of machine and process qualification trials, it is sometimes reasonable to establish a benchmark capability. A typical bench mark is c_{pk} = 1.33 which will make non conforming units unlikely in many situation (Hoffer 1985).

* Assisting product designers / developers in selecting or modifying a process (Product and Process design).

* Assisting in establishing an interval between sampling for process control (Quality Control Planning) – Within a department or plant it is often useful to monitor continuous improvement, which can be accomplished by observing the changing distribution of process capabilities. For example, if there were 10% of processes with capabilities between 1 and 1.33in a month and some of these improved to between 1.33 to 1.67 the next month, improvement has occurred. This distributional shift can easily be monitored.

* Specifying performance requirement for new equipment (Purchase Quality Control).

* Selecting between competing vendors (Vendor Sourcing).

* Planning the sequence of production processes when there is an interactive effect of processes on tolerances (Manufacturing Planning / Production Planning)

* Reducing the variability in a manufacturing process (Process Control) – For each characteristic, it is meaningful to compare C_p and C_{pk}. If Cpk is too low, then C_p must be examined to determine whether the variability is unacceptably high. If Cp is close to C_{pk}, then process location is not a problem. The indices C_{pu}, C_{pl} and k provide an assessment of how close the process mean is from the target mean.

In the electronics or microelectronics manufacturing industry Process capability measures convey critical information regarding percentage of conforming items, meeting product design speciation limits, which is a basic criterion used for judging whether the products are reliable from manufacturing perspective. A high value of PCI implies a high quality of the product. The most important application of Process Capability Analysis is in measurement Phase, and in semiconductor industry for quality control. Other use of Process Capability Indices in the supplier certification process; in optimization of multi response problems for batch manufacturing processes and quality computation model of complex assembling process using multivariate process capability index.

4. FUTURE SCOPE OF THE WORK AND CONCLUSION

Properties of the PCI index, suggested by various researchers are examined under different distributional assumptions. The obtained results offer a useful approach for measuring process capabilities on the basis of quantitative aspects since none of the most broadly used capability indices can be used in connection with discrete type of data despite the fact that they are quite frequently encountered in process control. The study of the properties of this index on the basis of such data under different distributional assumptions, for either continuous or

discrete processes that usually arise in applications, would be an interesting issue for further research.

Often the quality of a process is determined by several correlated univariate variables. Various different multivariate process capability indices (MPCI) have been developed for such a situation, but confidence intervals or tests have been derived for only a handful of these. Our objective is to develop new PCI and a decision procedure, based on a case study from the industry, to be used to decide whether a process can be capable or not at a stated significance level.

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REFERENCES

[1] Bender A. Statistical tolerancing as it relates to quality control and the designer. Automotive Division Newsletter of ASQC(1975) .

[2] Ekvall, D.N. Juran, M.J., Manufacturing planning. In: Juran, J.M. (Ed.), Quality Control Handbook, third ed. McGraw-Hill, New York 1974.

[3] V.E. Kane Process capability indices J. Qual. Technol., 18 (1986), pp. 41–52.

[4] Y.M. Chou, D.B. Owen On the distributions of the estimated process capability indices Comm. Statist. Theory Methods, 18 (1989), pp. 4549–4560.

[5] L.K. Chan, Z. Xiong, D. Zhang On the asymptotic distribution of some process capability indices Comm. Statist. Theory Methods, 19 (1990), pp. 11–18.

[6] S.E. Ahmed, A.K.M.E. Saleh Improved nonparametric estimation of location vector in a multivariate regression model Nonparametric Statist., 11 (1999), pp. 51–78.

[7] Y.S. Chang, I.S. Choi, D.S. Bai Process capability indices for skewed populations Qual. Reliab. Engng. Internat., 18 (2002), pp. 383–393.

[8] J.A. Clements Process capability calculations for non-normal distributions Qual. Progr., 22 (1989), pp. 98–100.

[9] S. Kotz, N.L. Johnson Process Capability Indices Chapman and Hall, London (1993).

[10] Kotz, S., Johnson, N.L., 2002. Process Capability Indices—A review, 1992–2000. Chapman and Hall, London, J Qual. Technol. 34, 1–19.

[11] Krishnamoorthi, K.S., Khatwani, S. A capability index for all occasions. Annual Quality Congress Proceedings, ASQ, Milwaukee, pp. 77–81(2000).

[12] P.C. Nahar, L.S. Zimmer Assessment of a capability index sensitive to skewness Qual. Reliab. Engng. Internat., 17 (2001), pp. 233–241.

[13] R. Noorossana Process capability analysis in the presence of autocorrelation Qual. Reliab. Engng. Internat., 18 (2002), pp. 75–77.

[14] W.L. Pearn, G.H. Lin On the reliability of the estimated incapability index Qual. Reliab. Engng. Internat., 17 (2001), pp. 279–290.

[15] W.L. Pearn, S. Kotz, N.L. Johnson Distributional and inferential properties of process capability indices J. Qual. Technol., 24 (1992), pp. 216–231.

[16] W.L. Pearn, G.H. Lin, K.S. Chen Distribution and inferential properties of the process accuracy and process precision indices Comm. Statist. Theory Methods, 27 (1998), pp. 985–1000.

[17] R.N. Rodriguez Recent developments in process capability analysis J. Qual. Technol., 24 (1992), pp. 176–187.

[18] S.E. Sommerville, D.C. Montgomery Process capability indices and non-normal distributions Qual. Engng., 9 (1996), pp. 305–316.

[19] Bothe DR (2002). Statistical reason for the 1.5s shift. Qual. Eng.,14(3): 479-487.

[20] Chan LK, Cheng SW and Spiring FA (1988). A new measure of process capability. J. Qual. Tech., 20(3): 162-175.

[21] Chen KS. Pearn WL. An application of non-normal process capability indices. Qual. Reliab. Eng. Int., (1997) 13: 355-360.

[22] Choi KC, Nam KH, Park DH. Estimation of capability index based on bootstrap method. Microelectron Reliab, (1996) 36(9): 141-153.

[23] Erto P, Pallotta G. A New Control Chart for Weibull Technological Processes. Qual. Tech. Quant. Manag., (2007) 4(4): 553-567.

[24] Hsu YC, Pearn WL, Pei CW. Capability adjustment for Gamma processes with mean shift consideration in implementing six sigma program. Eur. J. Oper. Res., (2008) 191(2): 517-529.

[25] Lu XM, Peng NF. The approximation distribution for the sum of independent and identical Weibull distributions. National Chiao Tung University, Taiwan(2003).

[26] Nichols MD, Padgett WD .A bootstrap control chart for Weibull percentiles. Qual. Reliab. Eng. Int., (2006). 22: 141-151.

[27] Shore H. A new approach to analyzing non-normal quality data with application to process capability analysis. Int. J. Prod. Res., (1998) 36(7):1917-1933